Preprint typeset in JHEP style - HYPER VERSION

Higher Spin Gauge Fields Interacting with Scalars: The Lagrangian Cubic Vertex

Angelos Fotopoulos

Dipartimento di Fisica Teorica dell'Università di Torino and INFN
Sezione di Torino, via P. Giuria 1, I-10125 Torino, Italy

Nikos Irges, Anastasios C. Petkou and Mirian Tsulaia

Department of Physics, University of Crete, 71003 Heraklion, Crete, Greece

Abstract: We apply a recently presented BRST procedure to construct the Largangian cubic vertex of higher-spin gauge field triplets interacting with massive free scalars. In flat space, the spin-s triplet propagates the series of irreducible spin-s, $s-2$,..,0/1 modes which couple independently to corresponding conserved currents constructed from the scalars. The simple covariantization of the flat space results is not enough in AdS, as new interaction vertices appear. We present in detail the cases of spin-2 and spin-3 triplets coupled to scalars. Restricting to a single irreducible spin-s mode we uncover previously obtained results. We also present an alternative derivation of the lower spin results based on the idea that higher-spin gauge fields arise from the gauging of higher derivative symmetries of free matter Lagrangians. Our results can be readily applied to holographic studies of higher-spin gauge theories.

Contents

1. Introduction

Constructing consistent interactions for higher-spin (HS) gauge fields is an old problem (see [1] for recent reviews). A crucial step for its resolution in AdS spaces was taken some years ago by Fradkin and Vasiliev [2] (see also [3]). Since then, the study of HS gauge theories has enjoyed a remarkable renaissance and a wealth of new and interesting results have appeared [4]–[20] (see also [21]– [25] for the earlier work).

One of the approaches to the interaction of HS gauge fields is based on the BRST-like methods (see e.g., [26] for a review). This is particularly appealing as it resembles similar constructions in string field theory [27]– [28]. In particular a model of interacting massless HS gauge fields can be obtained using a cubic vertex of the open string theory [29]. However in the general case of interacting massless HS fields there is no analog of overlap conditions such as that is present in the Open String Field Theory and therefore one has to consider a general polynomial of the corresponding matter and ghost oscillators.

In [30] a systematic method for the construction of the general cubic coupling of any three HS gauge fields in flat and AdS spaces based on the triplet construction [31]–[33] was presented. In the present note we apply our method to the simplest case; the interaction between one HS triplet and two massive free scalars. Despite its apparent simplicity this is still a highly non-trivial case since it requires the construction of an infinite number of conserved currents, made out of scalars, that couple properly to HS gauge fields. This is also an important case since, as we will see in Section 7, it elucidates the emergence of HS gauge fields via the gauging of higher derivative symmetries of free matter Lagrangians (see [34] for the discussion about self-adjoint operators in the gauging of HS symmetries). In a holographic setting, our results imply the existence of an infinite set of Ward identities involving scalar operators in boundary CFTs that are dual to HS gauge theories in AdS spaces.

The paper is organized as follows:

In Section 2 we briefly review our general method of constructing free and interacting Lagrangians for HS gauge fields [30]–[32]. In Section 3 we present the general result for the Lagrangian cubic interaction vertices between one triplet and two massive free scalars in flat space. We note the emergence of a pattern; the irreducible spin-$s, s-2, ..., 0/1$ modes propagated by the spin-s triplet couple independently to corresponding conserved currents constructed from the scalars. In Section 4 we outline how we obtain the general result for AdS. The simple flat space pattern is no more valid and new interaction vertices appear at order $1/L^2$. To keep our presentation clear we relegate the explicit lengthy expressions in the Appendix. In Section 5 we present the explicit formulae for the spin-2 and spin-3 cases in flat and AdS spaces. In Section 6 we present the results for the cubic vertex of irreducible HS gauge fields interacting with massive free scalars and show that our formulae reproduce known past results [35]–[37]. Extensive discussions of conformal HS currents were presented also in [36, 38, 39]. In Section 7 we re-derive the spin-2 and spin-3 vertices in flat and AdS spaces by an alternative method based on the idea that HS gauge fields arise from the gauging of high derivative symmetries of free matter Lagrangians [40]–[41]. This procedure opens up the possibility for explicit study of the HS gauge symmetry

acting on scalars, however we leave this interesting idea for a future work. Section 8 contains a summary and the outlook of our work. Important notation and some lengthy formulae appear in the five Appendices.

2. The BRST approach to the HS cubic vertex

In this Section we review the BRST approach to constructing the cubic interaction of HS gauge fields. More details can be found in [32], [30]. We consider an AdS space with radius $R_{AdS} = L$; the corresponding equations for flat space are simply obtained by setting the AdS curvature to zero.

The full interacting Lagrangian can be written as [27] – [28]

$$L = \sum_{i=1,2,3} \int dc_0^i \langle \Phi_i | \, Q_i \, |\Phi_i\rangle \; + g(\int dc_0^1 dc_0^2 dc_0^3 \langle \Phi_1 | \langle \Phi_2 | \langle \Phi_3 || V\rangle + h.c) \,, \tag{2.1}$$

where $|V\rangle$ is the cubic vertex and g is a dimensionless coupling constant.[1] To describe the cubic interaction of HS gauge fields we introduced three vectors $|\Phi_i\rangle$ $(i = 1, 2, 3)$ associated to three - generally different - Fock spaces spanned by the oscillators

$$[\alpha^i_\mu, \alpha^{j,+}_\nu] = \delta^{ij} g_{\mu\nu}, \quad \{c^{i,+}, b^j\} = \{c^i, b^{j,+}\} = \{c^i_0, b^j_0\} = \delta^{ij} \; . \tag{2.2}$$

The vacuum in each one of the Fock spaces is defined as

$$c|0\rangle \;=\; 0 \;, \qquad b|0\rangle \;=\; 0 \;, \qquad b_0|0\rangle \;=\; 0 \;, \qquad \alpha^\mu|0\rangle = 0 \,. \tag{2.3}$$

Each of the fields $|\Phi_i\rangle$ (so called "triplets") has the form

$$|\Phi\rangle = |\phi\rangle + c^+ \; b^+ \; |D\rangle + c_0 \; b^+ \; |C\rangle \,, \tag{2.4}$$

with

$$|\phi\rangle = \frac{1}{s!} \, h_{\mu_1 \ldots \mu_s}(x) \alpha^{\mu_1 +} \ldots \alpha^{\mu_s +} \; |0\rangle, \tag{2.5}$$

$$|D\rangle = \frac{1}{(s-2)!} \, D_{\mu_1 \ldots \mu_{s-2}}(x) \alpha^{\mu_1 +} \ldots \alpha^{\mu_{s-2} +} \; |0\rangle \;, \tag{2.6}$$

$$|C\rangle \;=\; \frac{-i}{(s-1)!} \, C_{\mu_1 \ldots \mu_{s-1}}(x) \alpha^{\mu_1 +} \ldots \alpha^{\mu_{s-1} +} \; |0\rangle \tag{2.7}$$

and the identically nilpotent BRST charge [32] in each of the Fock spaces has the form

$$\begin{aligned} Q = c_0\Big(l_0 \, + \, & \frac{1}{L^2}(N^2 - 6N + 6 + \mathcal{D} \, - \, \frac{\mathcal{D}^2}{4} - 4M^+ M + c^+ b(4N - 6) \\ & + b^+ c(4N - 6) + 12 c^+ b b^+ c - 8 c^+ b^+ M + 8 M^+ b c)\Big) \\ & + c^+ l + c l^+ - c^+ c b_0 \end{aligned} \tag{2.8}$$

[1]Each term in the Lagrangian (2.1) has length dimension $-\mathcal{D}$. This requirement holds true for each space-time vertex contained in (2.1) after multiplication by an appropriate power of the length scale of the theory, as discussed in [30].

with

$$l_0 \;=\; g^{\mu\nu}p_\mu p_\nu\,, \quad l=\alpha^\mu p_\mu, \quad l^+=\alpha^{\mu+}p_\mu\,, \tag{2.9}$$

and

$$N \;=\; \alpha^{\mu+}\alpha_\mu \;+\; \frac{\mathcal{D}}{2}, \quad M=\frac{1}{2}\alpha^\mu\alpha_\mu\,. \tag{2.10}$$

The momentum operator p_μ is defined as [42]

$$p_\mu \;=\; -\,i\,\left(\nabla_\mu+\omega_\mu^{ab}\,\alpha^+_{\;a}\,\alpha_{\;b}\right)\,, \;\; \alpha^a=e^a_\mu\alpha^\mu\,, \tag{2.11}$$

where e^a_μ and ω_μ^{ab} are the vierbein and the spin connection of AdS and ∇_μ is the AdS covariant derivative. The Lagrangian is invariant up to terms of order g^2 under the nonabelian gauge transformations

$$\delta|\Phi_i\rangle = Q_i|\Lambda_i\rangle - g\int dc_0^{i+1}dc_0^{i+2}[(\langle\Phi_{i+1}|\langle\Lambda_{i+2}|+\langle\Phi_{i+2}|\langle\Lambda_{i+1}|)|V\rangle]+O(g^2)\,, \tag{2.12}$$

with gauge transformation parameters

$$|\Lambda_i\rangle = b^{i,+}|\lambda^i\rangle = \frac{i}{(s-1)!}\lambda^i_{\mu_1\mu_2\ldots\mu_{s-1}}(x)\alpha^{i,\mu_1+}\alpha^{i,\mu_2+}\ldots\alpha^{i,\mu_{s-1}+}b^+|0\rangle\,, \tag{2.13}$$

provided that the vertex satisfies the BRST invariance condition

$$\tilde{Q}|V\rangle=0 \;\;,\quad \tilde{Q}=\sum_i Q_i. \tag{2.14}$$

The vertex operator $|V\rangle$ has ghost number $+3$ and its structure is

$$|V\rangle = V|-\rangle_{123}, \quad |-\rangle_{123}=c_0^1c_0^2c_0^3\;|0\rangle_1\otimes|0\rangle_2\otimes|0\rangle_3\,, \tag{2.15}$$

where V is an unknown function of the rest of the oscillators with ghost number zero. Note that equation (2.14) determines the vertex up to $\tilde{Q}$-exact terms

$$\delta|V\rangle=\tilde{Q}|W\rangle\,, \tag{2.16}$$

where $|W\rangle$ is an operator with total ghost number $+2$. The BRST-exact terms correspond to the ones which can be obtained by field redefinitions from the free Lagrangian. To simplify the analysis of equation (2.14) we define the operator

$$\tilde{N}=\alpha^{\mu,i+}\alpha^i_\mu+b^{i,+}c^i+c^{i,+}b^i\,. \tag{2.17}$$

This operator commutes with the BRST charges Q_i. This means that the vertex can be expanded in a sum of terms, each with fixed eigenvalues K of the operator $\tilde{N}$ as

$$|V\rangle=\sum_K|V(K)\rangle\,. \tag{2.18}$$

Therefore, equation (2.14) can be split into the set of algebraic equations

$$\sum_i Q_iV(K)=0 \tag{2.19}$$

for each value of K. The systematic solution of these equations determines the type the of cubic vertex.

3. The cubic vertex in flat space

In this section we will use our general approach to construct the cubic coupling of an HS triplet with two scalars in flat space. The corresponding coupling for irreducible HS fields has been discussed [44] and more recently in [35, 37]. The merit of considering the triplet is twofold. Firstly, our general BRST construction is systematic, can be straightforwardly generalized to $AdS_{\mathcal{D}}$ and is relevant for string field theory constructions. Secondly, our construction gives rise to the idea that HS gauge fields arise from the gauging of higher derivative symmetries of free matter Lagrangians.

To proceed, we put the triplet of spin s in Fock space 3 and the two scalars ϕ_1 and ϕ_2 into Fock spaces 1 and 2. Therefore, the matter and complex ghost oscillators appear only in Fock space 3. This allows us to write down the most general polynomial in terms of the expansion in ghost variables as (see Appendix A for the definition of the various operators used below)

$$\langle V| = {}_{123}\langle -|\Big\{X^1 + X^2_{33}\gamma^{33} + X^3_{3j}\beta^{3j}\Big\}\,, \quad j=1,2,3\,, \tag{3.1}$$

where

$$X^1 = X^1_{n_1,n_2,n_3;m_3,k_3;p_3,}\ (l_0^1)^{n_1}(l_0^2)^{n_2}(l_0^2)^{n_3}(l^3)^{m_3}(I^3)^{k_3}(M^{33})^{p_3}\,, \tag{3.2}$$

and with the same expansions for the coefficients X^2_{33} and X^3_{3j}. We will apply the field redefinition (FR) scheme outlined in [30] in order to eliminate the l_0^i from all three matrix elements, and the l^3 dependence from X^1 and X^2. This amounts to removing the indices n_1, n_2, n_3 from the expressions of X^1, X^2_{33} and X^3_{3j}. Then, the BRST invariance condition (2.14) simplifies to

$$\begin{aligned} {}_{123}\langle -|(I_3)^{k_3}\ (l_{33})^{m_3}\ (M_{33})^{p_3}\Big[&-X^3_{31;k_3,m_3,p_3}\ l_0^{11} - X^3_{32;k_3,n_3,p_3}\ l_0^{22} - \\ &X^3_{33;k_3,n_3,p_3}l_0^{33} + \delta_{m_3,0}(l^+_{33}\ X^1_{k_3,0,p_3} - l_{33}\ X^2_{33;k_3,0,p_3})\Big] = 0\,. \end{aligned} \tag{3.3}$$

We can solve (3.3) to derive

$$\begin{aligned} X^3_{31;0,k_3-1,p_3} &= k_3 X^1_{0,k_3,p_3}\,, \\ X^3_{32;0,k_3-1,p_3} &= -k_3 X^1_{0,k_3,p_3}\,, \\ X^2_{33;0,k_3,p_3-1} &= -p_3 X^1_{0,k_3,p_3}\,, \\ X^3_{33;m_3,k_3,p_3} &= 0\,, \\ X^3_{ij;m_3\neq 0;k_3,p_3} &= X^3_{33;m_3\neq 0;k_3,p_3} = X^1_{m_3\neq 0;k_3,p_3} = 0\,. \end{aligned} \tag{3.4}$$

For a triplet of spin-s the solution takes the form (we drop the m_3 index from now on):

$$\begin{aligned} \langle V| = {}_{123}\langle -|\Big\{&X^1_{k_3,p_3} - (p_3+1)X^1_{k_3,p_3+1}\gamma^{33,+} \\ &+(k_3+1)X^1_{k_3+1,p_3}\beta^{31,+} - (k_3+1)X^1_{k_3+1,p_3}\beta^{32,+}\Big\}(I^{+,3})^{k_3}(M^{+,33})^{p_3}\,, \end{aligned} \tag{3.5}$$

with

$$s = 2p_3 + k_3, \quad X^1_{k_3,p_3} = 2^p_3\ C_{s,p_3} \quad k_3 = 0,1,..,s\,. \tag{3.6}$$

Using the "momentum conservation" condition $p^1_\mu + p^2_\mu + p^3_\mu = 0$ and the solution above we can write the gauge transformation rules for scalars (2.12) in the form

$$\delta\phi_a = g \sum_{p=0}^{[\frac{s-1}{2}]} \xi^{ab}\ (s-2p)\ C_{s,p} \sum_{r=0}^{s-1-2p} \begin{pmatrix} s-1-2p \\ r \end{pmatrix}\ 2^r$$

$$(\partial^{\mu_{r+1}} \dots \partial^{\mu_{s-1-2p}} \lambda^{[p]}_{\mu_1 \dots \mu_{s-1-2p}})\ (\partial^{\mu_1} \dots \partial^{\mu_r} \phi_b) \tag{3.7}$$

$$\xi^{12} = (-1)^{s-1}\xi^{21} = -1,\ \xi^{11} = \xi^{22} = 0 \quad a, b = 1, 2\,, \tag{3.8}$$

where $C_{s,p}$ are arbitrary parameters. Note that the gauge transformations of the scalars are nonabelian, whereas the gauge transformations of the fields in the spin-s triplet remain abelian

$$\delta h_{\mu_1 \dots \mu_s} = \partial_{\{\mu_1} \lambda_{\mu_2 \dots \mu_s\}}, \quad \delta C_{\mu_1 \dots \mu_{s-1}} = \Box \lambda_{\mu_1 \dots \mu_s}, \quad \delta D_{\mu_1 \dots \mu_{s-2}} = \partial^\mu \lambda_{\mu, \mu_1 \dots \mu_{s-2}} .. \tag{3.9}$$

At this point we should emphasize that we have not imposed any symmetry between the scalars ϕ_a. It is obvious though from (3.8) that for HS triplets with s odd ($s = 2k+1$) one needs at least two *different* real scalars (or alternatively one complex scalar) to have a non-zero coupling [2]. The simple example of this is the $s = 1$ case where we find linearized scalar electrodynamics. For even s HS triplets ($s = 2k$) the condition (3.8) gives no obstruction to coupling with a single real scalar.

Finally, after some rather lengthy rearrangement we obtain the cubic interaction terms in the Lagrangian as

$$\begin{aligned}
\mathcal{L}_{s00} = & \int d^d x\ \Big\{ \sum_{p=0}^{[\frac{s}{2}]} C_{s,p}\ \mathcal{W}^{[p]}_{\mu_1 \dots \mu_{s-2p}} \times \\
& \sum_{r=0}^{s-2p} \begin{pmatrix} s-2p \\ r \end{pmatrix}\ (-1)^r\ (\partial^{\mu_1} \dots \partial^{\mu_r} \phi_1)\ (\partial^{\mu_{r+1}} \dots \partial^{\mu_{s-2p}} \phi_2)\ +\ h.c. \Big\} \\
= & \int d^d x \sum_{p=0}^{[\frac{s}{2}]} C_{s,p}\ \mathcal{W}_p \cdot J_{s-2p}\ +\ h.c.\,,
\end{aligned} \tag{3.10}$$

where we have used the binomial coefficients $\begin{pmatrix} n \\ m \end{pmatrix}$ and $[\frac{s}{2}]$ is the integer part of $\frac{s}{2}$. $\mathcal{W}_p$ is defined in [31]

$$\mathcal{W}_p = h^{[p]} - 2p\ D^{[p-1]}\,,\ \ \delta\mathcal{W}_p = \partial\Lambda^{[p]}\,, \tag{3.11}$$

and

$$J_{s-2p} = \sum_{r=0}^{s-2p} \begin{pmatrix} s-2p \\ r \end{pmatrix}\ (-1)^r\ (\partial^{\mu_1} \dots \partial^{\mu_r} \phi_1)\ (\partial^{\mu_{r+1}} \dots \partial^{\mu_{s-2p}} \phi_2)\,. \tag{3.12}$$

[2]Indeed setting $\phi_1 = \phi_2$ in (3.7) and taking into account (3.8) leads into an inconsitency for s-odd while it is allowed for s-even.

The fields $\mathcal{W}_p$ define a chain of lower spin fields contained in the triplet as can easily be seen from their gauge transformation properties (3.11). They are rank $s-2p$ symmetric tensors. Hence, the currents of (3.12) are also symmetrized and by virtue of the transformation properties (3.11) are conserved. We see a pattern emerging: given a general free scalar Lagrangian we can construct the series of spin-$s, s-2, ..., 0/1$ symmetric conserved currents (3.12) that couple properly to a spin-s triplet. In Section 7 we will try to understand the deep reason for the existence of such currents.

4. The cubic vertex in AdS

In this section we present the construction of the cubic vertex of a triplet coupled to two massive free scalars in AdS space. In this case the calculations are rather more involved, nevertheless we will be able to obtain a relatively simple result for the vertex.

The vertex still has the form (3.1). Next, we choose an FR (field redefinition) scheme where one can eliminate all l_0^{ii} dependence in (A.3) and set $X^3_{33} = 0$ in (3.1). With this choice we are able to eliminate any l^i dependence from X^2_{33}. Therefore one has the expansion of the vertex

$$\begin{aligned}\langle V| = {}_{123}\langle -|(I_3)^{k_3}\ (l_{33})^{m_3}\ (M_{33})^{p_3}\Big\{ &X^1_{k_3,m_3,p_3} + X^2_{33;k_3,0,p_3}\ \gamma^{33} \\ &+ X^3_{31;k_3,m_3,p_3}\ \beta^{13} + X^3_{32;k_3,m_3,p_3}\ \beta^{23}\Big\}\,. \end{aligned} \tag{4.1}$$

The BRST invariance gives the equation

$$\begin{aligned}&{}_{123}\langle -|(I_3)^{k_3}\ (l_{33})^{m_3}\ (M_{33})^{p_3}\Big\{ -X^3_{31;k_3,m_3,p_3}\ (l_0^{11} - \frac{2\mathcal{D}-6}{L^2}) - \\ &X^3_{32;k_3,m_3,p_3}(l_0^{22} - \frac{2\mathcal{D}-6}{L^2}) + l^+_{33}X^1_{k_3,m_3,p_3} - l_{33}\delta_{m_3,0}X^2_{33;k_3,0,p_3}\Big\} = 0\,.\end{aligned} \tag{4.2}$$

In order to arrive at the equivalent of (3.4) we will have to commute all creation operators $\alpha_\mu^{+,3}$ to the left but we will also have to eliminate one of the three momenta i.e., $p^{\mu,3}$ using "momentum conservation". In flat space commutativity of momenta makes this a very easy task. In AdS this becomes rather involved due to the relation (C.1) (see also [30]). The rules one should apply are the following[3]:

- In order to use "momentum conservation" we move the operators p_3^μ to the far left of the expression. Then we substitute $p_1^\mu + p_2^\mu + p_3^\mu = 0$. For example, instead of writing

$$(l_{32})p_{\rho,3}(l_{32})p_2^\rho = p_{\rho,3}(l_{32})(l_{32})p_2^\rho\,, \tag{4.3}$$

which translates to

$$\int d^Dx\sqrt{-g}(\nabla^\rho\Lambda^{\mu\nu})(\nabla_\mu\nabla_\nu\nabla_\rho\phi^2)\phi^1 \tag{4.4}$$

[3] We set $L^2 = 1$ in what follows and restore it at the end by dimensional analysis.

we use

$$-(p^1_{\mu,}+p^2_{\mu})(l_{32})(l_{32})p^{\mu}_2\,, \tag{4.5}$$

which translates into

$$-\int d^Dx\sqrt{-g}\Lambda^{\mu\nu}[(\nabla_\mu\nabla_\nu\nabla^\rho\phi^2)(\nabla_\rho\phi^1)+(\nabla^\rho\nabla_\mu\nabla_\nu\nabla_\rho\phi^2)\phi^1]\,,. \tag{4.6}$$

- We will then use the equations of Appendix D to move any "non-contracted" momenta $p^i_\mu,\ i=1,2$ to the right until they form operators $l^{11}_0, l^{22}_0, l^{31}$ or l^{32} with operators $p^1_\mu,\ p^2_\mu$ or α^3_μ. In the present example we should push the operators p^1_μ and p^2_μ to the right until they form the operators l^{11}_0 and l^{12}_0 when combined with p^μ_2. This process will generate terms proportional to $\frac{1}{L^2}$. For the example above they are

$$-\int d^Dx\sqrt{-g}\frac{1}{L^2}[\Lambda^\mu_\mu(\Box\phi^2)\phi^1+(1-2\mathcal{D})\Lambda^{\mu\nu}(\nabla_\mu\nabla_\nu\phi^2)\phi^1] \tag{4.7}$$

  which can be seen from the second term in (4.6) when pushing the covariant derivative ∇_ρ to the right.

- We will commute creation oscillators to the left. In doing so we will once more generate non-contacted momenta $p^{\mu,i}$ $i=1,2$ which in turn have to be pushed to the right and will generate further $1/L^2$ terms as explained in the previous step.

- Finally the ordering rule is that all operators l^{11}_0, l^{22}_0 which do not commute with I^3, and l^3, are to be brought to the extreme right of the equation so that we compare operator expressions which have the same ordering.

This procedure results in some quite lengthy equations but our choice of FR scheme which has eliminated X^3_{33} simplifies the problem. Actually, we have to perform the manipulations described above only for the third and fourth terms in (4.2). For the fourth terms we just push the operator $p^{\mu,3}$ to the left, then use "momentum conservation" and then push the operator $p^{\mu,1}+p^{\mu,2}$ to the right as described above. The third term is the hardest one since it requires performance of commutators both among momenta and among oscillators.

The solution for generic triplet is quite involved but straightforward. In the Appendices we give some explicit formulae that are used in the manipulations described above. The full solution for the triplet will not be presented here. Instead, as an illustration of our technique we present the two simplest examples describing the interaction of spin-2 and spin-3 triplets with two massive free scalars.

5. Explicit examples

5.1 Spin-2 with two scalars

Since the oscillators $\alpha^{i,+}_\mu, c^{i,+}$ and $b^{i,+}$ occur only in the third Fock space we omit the index i for them in what follows. The field will will using are

$$|\Phi_1\rangle=\phi_1(x)|0\rangle,\quad |\Phi_2\rangle=\phi_2(x)|0\rangle, \tag{5.1}$$

$$|\Phi_3\rangle = (\frac{1}{2!}h_{\mu\nu}(x)\alpha^{\mu+}\alpha^{\nu+} + D(x)c^+b^+ - iC_\mu(x)\alpha^{\mu+}c_0^3 b^+)|0\rangle \,, \tag{5.2}$$

$$|\Lambda\rangle = i\lambda_\mu(x)\alpha^{\mu+}b^+|0\rangle \,. \tag{5.3}$$

The Lagrangian has the form

$$L = L_{free} + L_{int} \,, \tag{5.4}$$

$$\begin{aligned} L_{free} = & (\partial_\mu\phi_1)(\partial^\mu\phi_1) + (\partial_\mu\phi_2)(\partial^\mu\phi_2) + m^2(\phi_1^2+\phi_2^2) + (\partial_\rho h_{\mu\nu})(\partial^\rho h^{\mu\nu}) \\ & -4(\partial_\mu h^{\mu\nu})C_\nu - 4(\partial_\mu C^\mu)D - 2(\partial_\mu D)(\partial^\mu D) + 2C_\mu C^\mu \,, \end{aligned} \tag{5.5}$$

$$\begin{aligned} L_{int} = & C_{2,0}\ (h^{\mu\nu}(\partial_\mu\partial_\nu\phi_1)\phi_2 + h^{\mu\nu}(\partial_\mu\partial_\nu\phi_2)\phi_1 - 2h^{\mu\nu}(\partial_\mu\phi_1)(\partial_\nu\phi_2)) \\ & -C_{2,1}\ \phi_1\phi_2(h^\mu_\mu - 2D) \,. \end{aligned} \tag{5.6}$$

The relevant gauge transformations are

$$\delta\phi_1 = C_{2,0}\ (2\lambda^\mu\partial_\mu\phi_2 + \phi_2\partial_\mu\lambda^\mu) \,, \tag{5.7}$$

$$\delta\phi_2 = C_{2,0}\ (2\lambda^\mu\partial_\mu\phi_1 + \phi_1\partial_\mu\lambda^\mu) \,, \tag{5.8}$$

$$\delta h_{\mu\nu} = \partial_\mu\lambda_\nu + \partial_\nu\lambda_\mu, \quad \delta C_\mu = \Box\lambda_\mu, \quad \delta D = \partial_\mu\lambda^\mu \,. \tag{5.9}$$

According to our general construction, given in the section 2 we have obtained the cubic vertex which involves two different scalars and the triplet with higher spin 2. To obtain the interaction of a single scalar with the spin-2 field we need to set $\phi_1 = \phi_2$ [4]. It should also be noticed that for $\phi_1 = \phi_2$ (5.6) is equivalent to the linearized interaction of a scalar field with gravity as we explain in Section 7.1. and in particular in equations (7.4) and (7.6). The generalization for the coupling of a spin-2 triplet with an arbitrary number of scalar fields n goes in an analogous manner with the constants $C_{2,0}$ becoming $n \times n$ matrices. Similar things apply to the couplings of scalars with any HS triplet. For simplicity in what follows we will discuss only the two scalar case which the reader can generalize easily to the n scalar case.

In $AdS_{\mathcal{D}}$ we replace ordinary with covariant derivatives. There will be no other changes for the gauge transformation rules (i.e., for all fields $\delta_{AdS} = \delta$) (5.9) except for

$$\delta_{AdS}C_\mu = \delta C_\mu + \frac{1-\mathcal{D}}{L^2}\lambda_\mu \,, \tag{5.10}$$

The free Lagrangian is modified to include the standard AdS "mass -terms" of order $1/L^2$

$$\Delta L_{free} = -\frac{1}{L^2}(2h^\mu_\mu h^\nu_\nu - 16h^\mu_\mu D + 2h_{\mu\nu}h^{\mu\nu} + (4\mathcal{D}+12)D^2 + (2\mathcal{D}-6)\ (\phi_1^2+\phi_2^2)) \,. \tag{5.11}$$

The interaction part also changes and gets an additional piece

$$\Delta L_{int.} = C_{2,0}\ \frac{\mathcal{D}-1}{L^2}D\phi_1\phi_2 \,. \tag{5.12}$$

This is an additional interaction of the D scalar with a "spin-0" current.

[4] Notice that setting i.e. $\phi_2 = 0$ is meaningless since in our formalism that would mean to consider two Hilbert spaces, hence no cubic interaction vertex.

5.2 Spin-3 with two scalars

The spin-3 triplet is described by the field

$$|\Phi_3\rangle = (\frac{1}{3!}h_{\mu\nu\rho}(x)\alpha^{\mu+}\alpha^{\nu+}\alpha^{\rho+} + D_\mu(x)\alpha^{\mu+}c^+b^+ - \frac{i}{2}C_{\mu\nu}(x)\alpha^{\mu+}\alpha^{\nu+}c_0^3b^+)|0\rangle, \tag{5.13}$$

$$|\Lambda\rangle = \frac{i}{2}\lambda_{\mu\nu}(x)\alpha^{\mu+}\alpha^{\nu+}b^+|0\rangle\,. \tag{5.14}$$

The relevant scalar and gauge transformations are

$$\delta\phi_1 = 3i\ C_{3,0}\ (4\lambda^{\mu\nu}\partial_\mu\partial_\nu\phi_2 + \phi_2\partial_\mu\partial_\nu\lambda^{\mu\nu} + 4(\partial_\mu\phi_2)(\partial_\nu\lambda^{\mu\nu})) + i\ C_{3,1}\ \phi_2\lambda^\mu_\mu\,, \tag{5.15}$$

$$\delta\phi_2 = -3i\ C_{3,0}\ (4\lambda^{\mu\nu}\partial_\mu\partial_\nu\phi_1 + \phi_1\partial_\mu\partial_\nu\lambda^{\mu\nu} + 4(\partial_\mu\phi_1)(\partial_\nu\lambda^{\mu\nu})) - i\ C_{3,1}\ \phi_1\lambda^\mu_\mu\,, \tag{5.16}$$

$$\delta h_{\mu\nu\rho} = \partial_\mu\lambda_{\nu\rho} + \partial_\nu\lambda_{\mu\rho} + \partial_\rho\lambda_{\mu\nu}, \quad \delta C_{\mu\nu} = \Box\lambda_{\mu\nu}, \quad \delta D_\mu = \partial_\nu\lambda^\nu_\mu\,. \tag{5.17}$$

The free and interacting parts of the Lagrangian have the form

$$\begin{aligned} L_{free} = {} & (\partial_\mu\phi_1)(\partial^\mu\phi_1) + (\partial_\mu\phi_2)(\partial^\mu\phi_2) + m^2(\phi_1^2+\phi_2^2) + (\partial_\tau h_{\mu\nu\rho})(\partial^\tau h^{\mu\nu\rho}) \\ & -6(\partial_\rho h^{\mu\nu\rho})C_{\mu\rho} - 12(\partial_\mu C^{\mu\nu})D_\nu - 6(\partial_\mu D_\nu)(\partial^\mu D^\nu) + 3C_\mu C^\mu\,, \end{aligned} \tag{5.18}$$

$$\begin{aligned} L_{int.} = {} & i\ C_{3,0}\ (h^{\mu\nu\rho}\phi_1\partial_\mu\partial_\nu\partial_\rho\phi_2 - h^{\mu\nu\rho}\phi_2\partial_\mu\partial_\nu\partial_\rho\phi_1 - 3h^{\mu\nu\rho}(\partial_\mu\partial_\nu\phi_2)(\partial_\rho\phi_1) \\ & +3h^{\mu\nu\rho}(\partial_\mu\partial_\nu\phi_1)(\partial_\rho\phi_2)) \\ & +i\ C_{3,1}\ (h^{\mu\nu}_\nu - 2D^\mu)(\phi_1\partial_\mu\phi_2 - \phi_2\partial_\mu\phi_1) + h.c. \end{aligned} \tag{5.19}$$

Note that in this case, had we set $\phi_1 = \phi_2$ the interaction would vanish. Unlike the previous example for the case of an interacting triplet with the higher spin 3 with two scalars one cannot put the scalars ϕ_1 and ϕ_2 to be equal to each other so one needs a complex scalar in analogy with scalar electrodynamics. There is one more difference with respect to the previous example, namely when doing the deformation to the $AdS_\mathcal{D}$ case , apart from changing ordinary derivatives to covariant ones, both the Lagrangian and gauge transformation rules for scalars get deformed. Namely one has

$$\begin{aligned} \Delta L_{free} = {} & -\frac{1}{L^2}(6h^{\mu\rho}_\mu h^\nu_{\nu\rho} - 48h^{\mu\nu}_\mu D_\nu - (\mathcal{D}-3)h_{\mu\nu\rho}h^{\mu\nu\rho} + \\ & +18(\mathcal{D}+3)D^\mu D_\mu + (2\mathcal{D}-6)\ (\phi_1^2+\phi_2^2)) \end{aligned} \tag{5.20}$$

$$\Delta L_{int} = i\ C_{3,0}\ \frac{6\mathcal{D}}{L^2}\ D^\mu\ (\phi_1\nabla_\mu\phi_2 - \phi_2\nabla_\mu\phi_1) + h.c. \tag{5.21}$$

$$\delta_{AdS}\phi_1 = \delta_0\phi_1 - i\ C_{3,0}\ \frac{6}{L^2}\lambda^\mu_\mu\phi_2, \quad \delta_{AdS}\phi_2 = \delta\phi_2 + i\ C_{3,0}\ \frac{6}{L^2}\lambda^\mu_\mu\phi_1, \tag{5.22}$$

$$\delta_{AdS}C_{\mu\nu} = \delta C_{\mu\nu} + \frac{2(1-\mathcal{D})}{L^2}\lambda_{\mu\nu} + \frac{2}{L^2}g_{\mu\nu}\lambda^\rho_\rho. \tag{5.23}$$

6. Irreducible HS gauge field coupled to scalars in AdS

In this section we will study the much simpler case of an irreducible HS field in AdS coupled to two massive free scalars. From the triplet we readily find the irreducible spin-s gauge field upon imposing the conditions [31]

$$\mathcal{W}_p = \Lambda^{[p]} = 0, \qquad p \geq 1\,, \tag{6.1}$$

with the definitions (3.11). Note that this corresponds to setting the *compensator* field of [31] to zero. The corresponding Lagrangian formulation which gives the equation (6.1) as a equation of motion is available [43], [32] but since it is more complicated we add the equation (6.1) to triplet "by hand". The above conditions lead to traceless gauge parameters and double-traceless gauge fields, namely ($'$ denotes the trace)

$$\begin{aligned} \phi^{''} &= 0 \rightarrow (M_{33})^2\ |\phi_3\rangle = 0\,, \\ \lambda^{'} &= 0 \rightarrow M_{33}\ |\Lambda_3\rangle = 0\,. \end{aligned} \tag{6.2}$$

This simplifies the calculations since the only non-vanishing matrix elements are now $X^1_{s-2p,0,p=0,1}$, $X^3_{31;s-2p-1,0,p=0}$ and $X^2_{33;s-2p-2,0,p=0}$. The computation follows the steps of (4.2) but in this case we keep only terms up to the first power of $2M_{33} = X^2$ (see the Appendix E for details). The final result is

$$X^3_{31;s-1,0,0} = -X^3_{32;s-1,0,0} = sX^1_{s,0,0}\,, \tag{6.3}$$

$$X^2_{33,s-2,0,0} = -X^1_{s-2,0,1} + (\frac{s-1}{3}(2s^2 + (3\mathcal{D}-4)s - 6) - 2(s-2))X^1_{s,0,0}. \tag{6.4}$$

Based on the solution above the gauge transformation is given by the direct covariantization of (3.7) for $p = 0$. The cubic interaction is given by

$$\mathcal{L}_{s00} = C_{s,0} \int d^{\mathcal{D}}x\sqrt{g}\left\{\phi\cdot J^{\nabla}_s + \left(\frac{s-1}{6L^2}[2s^2 + (3\mathcal{D}-4)s - 6] - \frac{(s-2)}{L^2}\right)\phi^{'}\cdot J^{\nabla}_{s-2}\right\} + h.c. \tag{6.5}$$

where the currents J^{∇}_s and J^{∇}_{s-2} are the AdS covariantizations of the corresponding flat symmetric ones in (3.12), however only the *double-traceless* part of J_s and the *traceless* part of J_{s-2} survive. The interaction Lagrangian is a function of just one unknown normalization constant $C_{s,0}$.

Let us now discuss our result (6.5). Firstly, the flat space restriction of (6.5), (i.e. dropping the $1/L^2$ terms), implies that an irreducible spin-s HS gauge field couples to totally symmetric, conserved currents. These currents coincide with the ones obtained by Berends et. al. in [44]. For $s \geq 4$ only the double-traceless part of J_s survives.

In AdS, we define the modified current

$$J^{AdS}_s = J^{\nabla}_s + \left(\frac{s-1}{6L^2}[2s^2 + (3\mathcal{D}-4)s - 6] - \frac{(s-2)}{L^2}\right)\ g\ J^{\nabla}_{s-2}\,, \tag{6.6}$$

where g is the AdS metric. This current is not conserved but satisfies [5]

$$[\ \nabla\cdot J^{AdS}_s\ ]^{traceless} = 0\,. \tag{6.7}$$

[5]Remember the rule (6.2) applied to our computation in the traceless case. This means we dropped all terms proportional to any power of the metric g in the BRST computation

This condition implies that the covariantized flat spin-s current J_s^{∇} fails to be conserved by order $1/L^2$ terms.

A few more comments are in order here. In flat space, the conserved currents J_s are not the only ones that couple consistently to irreducible HS gauge fields. They can be modified, at will, by terms whose divergence gives zero upon contraction with the traceless gauge parameter λ. We fixed this freedom by imposing the conditions (6.1) i.e. setting *compensator* field to zero. In [35] an *additional* single zero trace condition was used for the conserved currents in order to uniquely fix the form of the higher-spin currents in flat space. This condition was generalized in AdS by [37]. In the above works, bulk conformal (or Weyl) invariance played a crucial role. We believe that our approach is more general since our HS gauge fields are coupled generically to massive scalars. It is also satisfying that our approach is naturally tied to BRST, as we believe that this is relevant for the application of our results to string theory.

7. An alternative derivation of the cubic couplings

In this section we present an alternative derivation of the cubic interaction vertices of free massive scalars with HS triplets. Although not yet under total control, the method is a generalization of the standard Noether gauging in field theory and is based on a surprisingly simple symmetry of the mass term in the free Lagrangian. We present here explicitly the spin-2 and spin-3 coupling and leave the discussion of the symmetry and the spin-s cases, with $s \geq 4$ for the future.

7.1 Free massive scalars coupled to the spin-2 triplet

Consider the Lagrangian for a massive free scalar field in flat space

$$\mathcal{L} = \frac{1}{2}(\partial^\mu \phi)(\partial_\mu \phi) + \frac{1}{2}m^2\phi^2 \,. \tag{7.1}$$

The idea is that the spin-2 triplet will emerge through the gauging of a symmetry of the above. The spin-2 triplet involves[6] the unconstrained symmetric field $h_{\mu\nu}$, the auxiliary scalar D, while the gauge parameter is the vector λ_μ. We are looking for a transformation of the fields ϕ that induces a change of (7.1) of the form

$$\delta\mathcal{L} = (\partial^\mu \lambda^\nu)T_{\mu\nu} = \frac{1}{2}\delta h^{\mu\nu}T_{\mu\nu} \,. \tag{7.2}$$

Had we found such a transformation, we would conclude; first that $T_{\mu\nu}$ is a conserved current and second that the interaction of the massive free scalars with the triplet is of the form

$$\mathcal{L}_{int} = -c_1\frac{1}{2}h^{\mu\nu}T_{\mu\nu} + c_2(h' - 2D)T \,. \tag{7.3}$$

c_1 and c_2 are arbitrary constants. Notice that gauge invariance cannot determine the "spin-0 current" T since the variation of the second term identically vanishes.

[6]In the section we always solve the algebraic equation for the fields C.

At this moment one might think we are just describing the gauging of diffeomorphisms. Indeed, using $\delta\phi = c_1 \epsilon_\mu \partial_\mu \phi$ one finds the canonical energy-momentum tensor

$$T^{can}_{\mu\nu} = (\partial_\mu \phi)(\partial_\nu \phi) - \eta_{\mu\nu} \mathcal{L} \,. \tag{7.4}$$

However, this is *not* what the BRST analysis gives, both for the transformation of scalars and for the conserved spin-2 current $T_{\mu\nu}$. Instead, we will look for a new principle that may fix the scalar field transformations not only for the spin-2 case but for HS as well.

To this effect, consider the most general infinitesimal (i.e. involving one derivative) transformation of scalars with gauge parameter the vector λ_μ

$$\delta\phi = c_1(\lambda^\mu \partial_\mu + \kappa(\partial \cdot \lambda))\phi \,. \tag{7.5}$$

Next we demand that (7.5) leaves invariant the mass term in (7.1) up to total derivatives (that do not play a role in the action). This uniquely determines the parameter $\kappa = 1/2$ and reproduces the corresponding transformation for scalars (5.7) or (5.8) (we can set $\phi_1 = \phi_2$ there). It is important to note that for $\kappa = 1/2$ (7.5) is *not* a Weyl transformation. Using (7.5) we can vary the kinetic term in (7.1) and we straightforwardly obtain the conserved current

$$T_{\mu\nu} = \frac{1}{2}\left((\partial_\mu \phi)(\partial_\nu \phi) - \phi \partial_\mu \partial_\nu \phi\right) \,, \tag{7.6}$$

which coincides with the corresponding conserved current in (5.6). It should be noted that (7.6) and the canonical energy-momentum tensor (7.4) give rise to the same conserved quantitites (energy and momentum) on-shell. Hence, we expect that the transformation (7.5) is actually equivalent to diffeomorphisms, the additional term being equivalent to a Field Redefinition in the BRST language.

Next we move to AdS. We covariantize the derivatives in the transformation (7.5) and we observe that it still leaves the mass term invariant up to total derivatives. However, the variation of the kinetic term is now different. Up to total derivatives we obtain

$$\delta\mathcal{L} = c_1 \frac{1}{2}(\nabla^\mu \lambda^\nu)\left((\nabla_\mu \phi)(\nabla_\nu \phi) - \phi \nabla_\mu \nabla_\nu \phi - \frac{\mathcal{D}-1}{4L^2}\eta_{\mu\nu}\phi^2\right) \,. \tag{7.7}$$

Hence, in AdS the coupling is modified as

$$-c_1 \frac{1}{2} h^{\mu\nu} T_{\mu\nu} \rightarrow -c_1 \left[\frac{1}{2} h^{\mu\nu} T^{\nabla}_{\mu\nu} - \frac{\mathcal{D}-1}{8L^2} D\phi^2\right] \,, \tag{7.8}$$

with $T^{\nabla}_{\mu\nu}$ denoting the covariantized current. This is in agreement with the result (5.12).

7.2 Free massive scalars coupled to the spin-3 triplet

In this case we must have two different scalars to begin with. The free Lagrangian is

$$\mathcal{L} = \frac{1}{2}(\partial^\mu \phi_i)(\partial_\mu \phi_i) + \frac{1}{2} m^2 \phi_i^2 \,, \quad i = 1, 2 \,. \tag{7.9}$$

The spin-3 triplet involves the symmetric tensor $h_{\mu\nu\rho}$, the vector D_μ, while the gauge parameter is the symmetric tensor $\lambda_{\mu\nu}$. Hence, under the scalar field transformation we expect that the free Lagrangian will vary as

$$\delta\mathcal{L} = q_1(\partial^\mu\lambda^{\nu\rho})T_{\mu\nu\rho} + q_2(\partial^\mu\lambda')T_\mu\,, \tag{7.10}$$

with q_1 and q_2 arbitrary constants. This would imply that the coupling to the triplet is

$$\mathcal{L}_{int} = -q_1\frac{1}{3}h^{\mu\nu\rho}T_{\mu\nu\rho} - q_2(h^{\mu\nu}_\nu - 2D_\mu)T_\mu\,. \tag{7.11}$$

Notice the presence of the spin-1 conserved current T_μ and the fact that q_1 and q_2 have different dimensions.

To construct the spin-3 current we seek first the most general scalar field transformation that involves the gauge parameter $\lambda_{\mu\nu}$ (and *not* its trace), that leaves the mass term invariant. A simple calculation gives the result[7]

$$\delta\phi_i = q_1\left(\lambda^{\mu\nu}\partial_\mu\partial_\nu + (\partial_\mu\lambda^{\mu\nu})\partial_\nu + B(\partial_\mu\partial_\nu\lambda^{\mu\nu})\right)\phi_j\epsilon_{ij}\,, \tag{7.12}$$

where we use the totally antisymmetric tensor ϵ_{ij}, $i,j = 1,2$. We note that the parameter B cannot be fixed by requiring invariance of the mass term. However, applying the transformation (7.12) to the kinetic term of (7.9) we get

$$\delta\mathcal{L} = q_1(\partial^\mu\lambda^{\nu\rho})\left((2B-1)(\partial_\mu\partial_\nu\phi_i)(\partial_\rho\phi_j) + B(\partial_\mu\phi_i)(\partial_\nu\partial_\rho\phi_j) + B(\partial_\mu\partial_\nu\partial_\rho\phi_i)\phi_j\right)\epsilon_{ij}. \tag{7.13}$$

Symmetrizing the current, in order to produce the $\delta h^{\mu\nu\rho}T_{\mu\nu\rho}$ term, we find $B = 1/4$, in agreement with the corresponding scalar fields transformations (5.15) and (5.16). The conserved spin-3 current we find is

$$T_{\mu\nu\rho} = \frac{1}{4}\left((\partial_\mu\partial_\nu\partial_\rho\phi_i)\phi_j - 3(\partial_\mu\partial_\nu\phi_i)(\partial_\rho\phi_j)\right)\epsilon_{ij}\,, \tag{7.14}$$

which coincides (up to an overall numerical factor) with (5.19).

Passing on to AdS, we first notice that the covariantization of the transformation (7.12) with $B = 1/4$ leaves the mass term invariant. However, the variation of the kinetic term is now altered. Alter a lengthy calculation we find that in AdS the coupling is modified as

$$-\,q_1\frac{1}{3}h^{\mu\nu\rho}T_{\mu\nu\rho} \to -q_1\left[\frac{1}{3}h^{\mu\nu\rho}T^\nabla_{\mu\nu\rho} - \frac{1}{2L^2}\left(\frac{2}{3}h^{\mu\nu}{}_\nu - (\mathcal{D}+1)D^\mu\right)T^\nabla_\mu\right]\,, \tag{7.15}$$

where the covariantized spin-1 current is

$$T^\nabla_\mu = (\nabla_\mu\phi_i)\phi_j\epsilon_{ij}\,. \tag{7.16}$$

This result is in agreement with the corresponding result of the BRST analysis (5.21) for $s = 3$. Indeed, a piece of (7.15) proportional to $h^\mu - 2D^\mu$ can be associated to a modification of the gauge transformation as in (5.22) and the remaining can be seen as a modification of the coupling. Nevertheless, a highly non-trivial check of (7.15) is that when we set $h'_\mu = 2D_\mu$ we get the result (6.6) which was gotten by a totally independent method. The term involving q_2 in the interaction (7.11) is simply modified by $T_\mu \to T^\nabla_\mu$.

[7]Note the similarity with the generalized Lie derivative obtained in the last of [8].

8. Summary and Outlook

We have applied the general BRST procedure of [30] to construct the cubic interaction Lagrangian vertex of HS triplets coupled to free massive scalars. Although this is the simplest possible case of HS interactions, it still involves considerable technical tasks. We were able to give closed expressions for the vertex in flat and AdS spaces, however, the AdS expressions are still quite involved.

The cubic vertex in flat space has an interesting structure. Namely, the spin-$s, s-2, ..., 0/1$ modes that are propagated by a spin-s triplet couple independently to corresponding conserved currents constructed from the scalars. In flat space these are the currents constructed long ago by Berends et. al. [44]. In AdS, the situation changes and generically both the gauge variations and the couplings are deformed by $1/L^2$ corrections. Although there might be a pattern for the AdS deformation we were not able to find it. We can pass to irreducible HS modes by simply setting the *compensator* fields [31] to zero. This gauge choice allows us to use the same symmetric and conserved currents found above for the coupling of scalars to irreducible HS gauge fields in flat space. Again, in AdS the currents are deformed by $1/L^2$ terms. We never use conformal or Weyl invariance in our construction as in the works [35, 37]. The detailed expressions for the spin-2 and spin-3 cases are given. The latter results are reproduced by an alternative method based on the idea that HS gauge fields arise via the gauging of higher-derivative symmetries of free matter Lagrangians.

There are many interesting applications and extensions of our work. Since we were able to couple HS gauge fields to massive scalars our results can be readily used in holography. In particular, an obvious implication of our results is the existence of an infinite set of Ward identities associated to composite scalar operators in conformal field theories dual to HS gauge theories[8] Also, our calculations are the first step towards the construction of the Lagrangian cubic vertex of HS gauge fields with spins $s \neq 0$ in AdS. The holographic interpretation of such a calculation will give the three-point functions of the energy momentum tensor and of an infinite set of higher spin conserved currents in the boundary CFT. This way we hope to understand the meaning of the parameters present in three-point functions of conserved currents of generic CFTs [45]. These issues will be studied in a forthcoming work.

Finally, a few words are reserved for the alternative derivation of the cubic couplings. A scalar field deformation in terms of a vector-like gauge parameter λ_μ is simply associated to diffeomorphisms. It would be extremely interesting to understand the origin of our higher-derivative scalar transformations, those that involve tensor gauge parameters, that keep the mass term invariant. It is conceivable that they indicate a broader structure for the underlying "spacetime", perhaps one that involves tensor coordinates. It would also be interesting to study the algebraic structure of our higher-derivative transformations. We intent to come back to these exciting questions in the near future.

[8]Similar ideas were discussed in [40].

Acknowledgments. We‘are grateful to X. Bekaert, N. Boulanger, O. Hohm and M. Vasiliev for useful comments. The work of A.F. is partially supported by the European Commission, under RTN program MRTN-CT-2004-0051004 and by the Italian MIUR under the contract PRIN 2005023102. The work of N. I. was partially supported by the "France-Greece Common Research Program in Science and Technology", K.A. 2342. The work of A. C. P. was partially supported by the PYTHAGORAS II Research Program K.A.2101, of the Greek Ministry of Higher Education. The work of M.T. was supported by the European Commission under RTN program MRTN-CT-2004-512194.

A. Basic Definitions

A.1 Definition of $|V\rangle$ and $|W\rangle$

We define two linearly independent combinations of variables with ghost number zero

$$\gamma^{ij,+} = c^{i,+}b^{j,+}, \qquad \beta^{ij,+} = c^{i,+}b_0^j\,. \tag{A.1}$$

Then, the most general expansion of the vertex in terms of ghost variables has the form

$$\begin{aligned}|V\rangle = \Big\{ & X^1 + X^2_{ij}\gamma^{ij,+} + X^3_{ij}\beta^{ij,+} + X^4_{(ij);(kl)}\gamma^{ij,+}\gamma^{kl,+} + X^5_{ij;kl}\gamma^{ij,+}\beta^{kl,+} + \\ & + X^6_{(ij);(kl)}\beta^{ij,+}\beta^{kl,+} + X^7_{(ij);(kl);(mn)}\gamma^{ij,+}\gamma^{kl,+}\gamma^{mn,+} + X^8_{(ij);(kl);mn}\gamma^{ij,+}\gamma^{kl,+}\beta^{mn,+} + \\ & + X^9_{ij;(kl);(mn)}\gamma^{ij,+}\beta^{kl,+}\beta^{mn,+} + X^{10}_{(ij);(kl);(mn)}\beta^{ij,+}\beta^{kl,+}\beta^{mn,+}\Big\}|-\rangle_{123}\ .\end{aligned} \tag{A.2}$$

The coefficients X^l depend only on operators α^{i+} and p^i, which means that they can be written as

$$\begin{aligned} X^l_{(\ldots)} &= X^l_{n_1,n_2,n_3;m_1,k_1,m_2,k_2,m_3,k_3;p_1,p_2,p_3,r_{12},r_{13},r_{23}(\ldots)} \\ & (l_0^1)^{n_1}\ldots(l^{+,1})^{m_1}(I^{+,1})^{k_1}\ldots(M^{+,11})^{p_1}\ldots(M^{+,12})^{r_{12}}\ldots \end{aligned} \tag{A.3}$$

where

$$l_0^{ij} = (l_0^{11}, l_0^{22}, l_0^{33}) = (l_0^1, l_0^2, l_0^3) \quad l^{ij} = (l^1, I^1, l^2, I^2, l^3, I^3), \tag{A.4}$$

$$I^1 = \alpha^{\mu,1}(p^2_\mu - p^3_\mu), \quad I^2 = \alpha^{\mu,2}(p^3_\mu - p^1_\mu), \quad I^3 = \alpha^{\mu,3}(p^1_\mu - p^2_\mu), \tag{A.5}$$

$$l^i = l^{ii}, \quad M^{ij} = \frac{1}{2}\alpha^{i\mu}\alpha^j_\nu \tag{A.6}$$

In a similar manner one has the following expansion for the operator $|W\rangle$

$$\begin{aligned}|W\rangle = \Big\{ & W^1_i b^{i,+} + W^2_i b^i_0 + W^3_{i;jk} b^{i,+}\gamma^{jk,+} + W^4_{i;jk} b^{i,+}\beta^{jk,+} + W^5_{i;jk} b^i_0\beta^{jk,+} + \\ & W^6_{i;(jk);(lm)} b^{i,+}\gamma^{jk,+}\gamma^{lm,+} + W^7_{i;jk;lm} b^{i,+}\gamma^{jk,+}\beta^{lm,+} + W^8_{i;(jk);(lm)} b^{i,+}\beta^{jk,+}\beta^{lm,+} + \\ & W^9_{i;(jk);(lm)} b^i_0\beta^{jk,+}\beta^{lm,+} + W^{10}_{i;(jk);(lm);pn} b^{i,+}\gamma^{jk,+}\gamma^{lm,+}\beta^{pn,+} + \\ & W^{11}_{i;jk;(lm);(pn)} b^{i,+}\gamma^{jk,+}\beta^{lm,+}\beta^{pn,+} + W^{12}_{i;(jk);(lm);(pn)} b^{i,+}\beta^{jk,+}\beta^{lm,+}\beta^{pn,+}\Big\}|-\rangle_{123}\,.\end{aligned} \tag{A.7}$$

Alternatively, one can expand in terms of the operators $l^{ij+} = \alpha^{\mu i,+}p^j_\mu$ and $l^{ij}_0 = p^{\mu i}p^j_\mu$ but one has to bear in mind that not all of these are independent due to the momentum conservation law $p^1_\mu + p^2_\mu + p^3_\mu = 0$ (see [30] for more details).

B.2 Definition of the F, G functions and their properties

The following identities hold:

$$\sum_{u=0}^{[\frac{n-1}{2}]} \binom{n}{2u+1} a^{n-2u}\ x^{2u} = \frac{a}{2x}\ [(a+x)^n - (a-x)^n] \tag{B.1}$$
$$\sum_{u=0}^{[\frac{n}{2}]} \binom{n}{2u} a^{n-2u}\ x^{2u} = \frac{1}{2}\ [(a+x)^n + (a-x)^n]$$
$$\sum_{k=0}^{n} a^k (a+x)^{n-k} = -\frac{1}{x}(a^{n+1} - (x+a)^{n+1})$$

Then we define

$$\begin{aligned} F(n,0,Y,X) &= \sum_{k=0}^{n} \sum_{u=0}^{[\frac{n-k}{2}]} \binom{n-k+1}{2u+1} Y^{n-2u+1}\ X^{2u} = \\ &= \frac{1}{2}\sum_{k=0}^{n+1} \binom{n+2}{k} Y^{k+1}\ X^{n-k}\ (1+(-1)^{n-k}) = \\ &= \frac{Y}{2X^2}\ [-2Y^{n+2} + (Y+X)^{n+2} + (Y-X)^{n+2}] \end{aligned} \tag{B.2}$$

and

$$\begin{aligned} F(n,1,Y,X) &= \sum_{k=0}^{n} \sum_{u=0}^{[\frac{n-k}{2}]} \binom{n-k}{2u} Y^{n-2u}\ X^{2u} = \\ &= \frac{1}{2}\sum_{k=0}^{n+1} \binom{n+1}{k} Y^{k}\ X^{n-k}\ (1+(-1)^{n-k}) = \\ &= \frac{1}{2X}\ [(Y+X)^{n+1} - (Y-X)^{n+1}] \end{aligned} \tag{B.3}$$

The function $F(n,\lambda,Y,X)$, $\lambda = 0,1$ as defined above has the property that it is an expansion in even powers of X.

Using (B.1) we can show the following identities:

$$\begin{aligned} &\sum_{k=0}^{n} Y^k\ F(n-k,0,Y,X) = \frac{Y}{X^2}\ [-(n+3)Y^{n+2} + F(n+2,1,Y,X)] \\ &\sum_{k=0}^{n} Y^k\ F(n-k,1,Y,X) = \frac{1}{Y}F(n,0,Y,X) \\ &\partial_Y F(n,0,Y,X) = \frac{1}{Y}F(n,0,Y,X) + (n+2)F(n-1,0,Y,X) \\ &\partial_Y F(n,1,Y,X) = (n+1)F(n-1,1,Y,X) \end{aligned} \tag{B.4}$$

$$
\begin{aligned}
\partial_{X^2} F(n,0,Y,X) &= -\frac{1}{X^2} F(n,0,Y,X) + \frac{Y}{2X^2} F(n,1,Y,X) \\
\partial_{X^2} F(n,1,Y,X) &= -\frac{1}{2X^2} F(n,1,Y,X) + (n+1)\frac{Y^n}{2X^2} + \frac{n+1}{2Y} F(n-2,0,Y,X)
\end{aligned}
$$

All of the above identities do not produce any negative powers of Y or X^2. This will be useful in Appendix D.

We finally define the following functions:

$$
\begin{aligned}
&G_e(n,0,Y,X) = \sum_{k=0}^{n} \sum_{u=0}^{[\frac{n-k}{2}]} \binom{n-k}{2u} \ F(n-2u,0,Y,X) X^{2u} = \\
&= \frac{Y}{2X^2}[-2Y^2 \ F(n,1,Y,X) + (Y+X)^2 \ F(n,1,Y+X,X) \\
&+(Y-X)^2 \ F(n,1,Y-X,X)] \\
&G_e(n,1,Y,X) = \sum_{k=0}^{n} \sum_{u=0}^{[\frac{n-k}{2}]} \binom{n-k}{2u} \ F(n-2u,1,Y,X) X^{2u} = \\
&= \frac{1}{2X}[(Y+X) \ F(n,1,Y+X,X) \\
&-(Y-X) \ F(n,1,Y-X,X)] \\
&G_o(n,0,Y,X) = \sum_{k=0}^{n} \sum_{u=0}^{[\frac{n-k}{2}]} \binom{n-k+1}{2u+1} \ F(n-2u,0,Y,X) X^{2u} = \\
&= \frac{Y}{2X^2}[-2Y \ F(n,0,Y,X) \\
&+(Y+X) \ F(n,0,Y+X,X) + (Y-X) \ F(n,0,Y-X,X)] \\
&G_0(n,1,Y,X) = \sum_{k=0}^{n} \sum_{u=0}^{[\frac{n-k}{2}]} \binom{n-k+1}{2u+1} \ F(n-2u,1,Y,X) X^{2u} \\
&= \frac{1}{2X}[(F(n,1,Y+X,X) - F(n,1,Y-X,X)]
\end{aligned}
\tag{B.5}
$$

The functions with arguments $Y+X$ and Y are related. Using:

$$
\begin{aligned}
(Y+X)^{n+2} &= \frac{X^2}{Y} \ F(n-1,0,Y,X) + X \ F(n,1,Y,X) + Y^{n+1} \\
(Y+X)^{n+2} &= \frac{X^2}{Y} \ F(n-1,0,Y,X) - X \ F(n,1,Y,X) + Y^{n+1}
\end{aligned}
\tag{B.6}
$$

one can write i.e.

$$
\begin{aligned}
&F(n,1,Y \pm X,X) = \pm\frac{1}{2X} \ [(Y \pm 2X)^{n+1} - Y^{n+1}] = \\
&= \pm\frac{2X}{Y} \ F(n-1,0,Y,2X) + F(n,1,Y,2X)
\end{aligned}
\tag{B.7}
$$

$$F(n,0,Y\pm X,X)=\frac{Y\pm X}{2X^2}\ [-2(Y\pm X)^{n+2}$$
$$+(Y\pm 2X)^{n+2}+Y^{n+2}]=\frac{Y\pm X}{X}\ F(n,0,Y,2X). \tag{B.8}$$

Using all of the above we can easily show for example that

$$G_e(n,0,Y,X)=\frac{Y}{2X^2}\ [-2Y^2\ F(n,1,Y,X)$$
$$+\ 4X^2\ F(n-1,0,Y,2X)+\ (Y^2+X^2)\ F(n,1,Y,2X)]$$

which makes it easy to expand in a single series expansion in terms of Y and X using (B.2,B.3). Finally we will define the following compact expression which will make the presentation of our results in the main text easier

$$\tilde{G}_e(n,\lambda,Y,X;a)=\sum_{k=0}^{n}\sum_{u=0}^{[\frac{n-k}{2}]}\binom{n-k}{2u}\ F(n-a-2u,\lambda,Y,X)X^{2u}=$$
$$=\ G_e(n-a,\lambda,Y,X)+\ \sum_{u=0}^{[\frac{n-a}{2}]}\ \sum_{k=0}^{a-1}\binom{n-k}{2u}\ F(n-a-2u,\lambda,Y,X)X^{2u} \tag{B.9}$$

and

$$\tilde{G}_o(n,\lambda,Y,X;a)=\sum_{k=0}^{n}\sum_{u=0}^{[\frac{n-k}{2}]}\binom{n-k+1}{2u+1}\ F(n-a-2u,\lambda,Y,X)X^{2u}=$$
$$=\ G_o(n-a,\lambda,Y,X)+\ \sum_{u=0}^{[\frac{n-a}{2}]}\ \sum_{k=0}^{a-1}\binom{n-k+1}{2u+1}\ F(n-a-2u,\lambda,Y,X)X^{2u} \tag{B.10}$$

C. Commutation relations

We wish to compute the commutators of $p^{\mu,i},\ i=1,2$ and $\alpha_\mu^{+,3}$ with strings of operators involving l^{3i} and M^{33}. We will use the following equation for a tensor $T_{\rho,\ldots}$:

$$D^{\mu\nu}T_{\rho,\ldots}=[p_\mu,p_\nu]T_{\rho,\ldots}=-\frac{1}{L^2}\ (g_{\nu\rho}T_{\mu\ldots}-g_{\mu\rho}T_{\nu\ldots})+\ldots \tag{C.1}$$

We will set $L^2=1$ and will reinstate it only at the end of our calculations based on dimensional analysis.

We start first with the momenta commutators. We drop the Fock index from the oscillators. The following computation holds:

$$S_\nu(n)=[\alpha^\mu D^2_{\mu\nu},(l_{32})^n]= \tag{C.2}$$
$$=\alpha^\mu\ \sum_{k=0}^{n-1}\ (l_{32})^k\ (\alpha_\nu p_{2,\mu}-\alpha_\mu p_{2,\nu})\ (l_{32})^{n-k-1}$$

We can then show by induction that:

$$S_\nu(n) = -2(1+\Theta_{n-2})M_{33}(l_{32})^{n-1}p_{2,\nu} + n\ \alpha_\nu (l_{32})^n + \\ +2M_{33}\sum_{k=0}^{n-2}\ \sum_{u=k}^{n-k-2}\ (l_{32})^{k+u}\ S_\nu(n-k-u-2) \tag{C.3}$$

Some straightforward manipulations show that

$$S_\nu(1) = \alpha_\nu l_{32} - 2M_{33}p_{2,\nu} \tag{C.4}$$
$$S_\nu(2) - l_{32}S_\nu(1) = \alpha_\nu (l_{32})^2 - 2M_{33}p_{2,\nu}$$
$$S_\nu(n) - l_{32}S_\nu(n-1) = \alpha_\nu (l_{32})^n + 2M_{33}\ \sum_{k=0}^{n-2}\ (l_{32})^k\ S_\nu(n-k-2)\ ,\ \ n \geq 3$$

where Θ_n is 0 for $n < 0$ and 1 otherwise.

Then by algebraic manipulations, induction and use of the formula:

$$\sum_{k=0}^{m}\ \binom{n+k}{n} = \binom{n+m+1}{n+1} \tag{C.5}$$

we arrive at the following solution:

$$S_\nu(n) = \tag{C.6}$$
$$\alpha_\nu\ \sum_{k=0}^{[\frac{n-1}{2}]}\ \binom{n}{2k+1}\ (l_{32})^{n-2k}(M_{33})^k + \sum_{k=0}^{[\frac{n}{2}]}\ \binom{n}{2k}\ (l_{32})^{n-2k}(M_{33})^k\ S_\nu(0)$$
$$-\sum_{k=0}^{[\frac{n-1}{2}]}\ \binom{n-1}{2k}\ (l_{32})^{n-1-2k}(M_{33})^{k+1}\ p_{2,\nu}$$
$$-\sum_{k=0}^{[\frac{n-2}{2}]}\ \binom{n-2}{2k}\ (l_{32})^{n-1-2k}(M_{33})^{k+1}\ p_{2,\nu}$$

It is easy to deduce that:

$$[p_{2\mu}, (l_{32})^n] = -\sum_{k=0}^{n-1}\ (l_{32})^k\ S_\nu(n-k-1) \tag{C.7}$$

Then using (C.6) and (B.2), (B.3) we arrive at the following relation:

$$[p_{2\mu}, (l_{32})^n] = -\frac{1}{L^2}\ [F(n-2,0,Y,\frac{X}{L})\ \alpha_\mu - F(n-2,1,Y,\frac{X}{L})\ X^2\ p_{2,\mu} \\ -F(n-3,1,Y,\frac{X}{L})\ Y\ X^2\ p_{2,\mu}] - F(n-1,1,Y,\frac{X}{L})\ S_\nu(0) \tag{C.8}$$

where we have reinstated the units L and also we have set $Y = l_{32}$ and $X^2 = 2M_{33}$. Note that although naively it might seem that X can appear in odd powers, therefore making no sense,

actually as we mentioned in Appendix A the function $F(n,\lambda,Y,X))$ always has an even argument in the variable X.

In a similar manner we can show that:

$$
\begin{aligned}
&[(l_{32})^n,\alpha_\mu^+] = n(l_{32})^{n-1}p_{2,\mu} - \frac{1}{L^2}\ [\frac{L^2Y}{X^2}\ (-n\ Y^{n-1} + F(n-1,1,Y,\frac{X}{L})\ )\ \alpha_\mu \\
&-(\frac{1}{Y}\ F(n-3,0,Y,\frac{X}{L}) + F(n-4,0,Y,\frac{X}{L}))\ X^2\ p_{2,\mu} \\
&+\frac{L^2}{Y}\ F(n-2,0,Y,\frac{X}{L})\ S_\mu(0)]
\end{aligned}
\tag{C.9}
$$

Finally we can compute the commutators of momenta and oscillators with $F(n,\lambda,Y,X)$:

$$
\begin{aligned}
&[p_\mu, F(n,0,Y,\frac{X}{L})] = -\frac{1}{L^2}\ \{\ \tilde{G}_o(n,0,Y,\frac{X}{L};1)\ \alpha_\mu - \\
&(\tilde{G}_o(n,1,Y,\frac{X}{L};1) + Y\tilde{G}_o(n,1,Y,\frac{X}{L};2))\ X^2\ p_\mu + L^2\tilde{G}_o(n,1,Y,\frac{X}{L};0)\ S_\mu(0)
\end{aligned}
\tag{C.10}
$$

$$
\begin{aligned}
&[p_\mu, F(n,1,Y,\frac{X}{L})] = -\frac{1}{L^2}\{\ \tilde{G}_e(n,0,Y,\frac{X}{L};2)\ \alpha_\mu - \\
&(\tilde{G}_e(n,1,Y,\frac{X}{L};2) + Y\tilde{G}_e(n,1,Y,\frac{X}{L};3))\ X^2\ p_\mu + L^2\tilde{G}_e(n,1,Y,\frac{X}{L};1)\ S_\mu(0)\}
\end{aligned}
\tag{C.11}
$$

$$
\begin{aligned}
&[F(n,0,Y,\frac{X}{L}),\alpha_\mu^+] = (\partial_Y F(n,0,Y,\frac{X}{L})\ (p_\mu + \frac{Y}{X^2}\alpha_\mu) + 2\partial_{X^2}F(n,0,Y,\frac{X}{L})\ \alpha_\mu \\
&-\frac{1}{L^2}\ \{\ \frac{L^2Y}{X^2}\tilde{G}_o(n,1,Y,\frac{X}{L};0)\ \alpha_\mu - (\frac{1}{Y}\tilde{G}_o(n,0,Y,\frac{X}{L};2) \\
&+\tilde{G}_o(n,0,Y,\frac{X}{L};3))\ X^2\ p_\mu + \frac{L^2}{Y}\tilde{G}_o(n,0,Y,\frac{X}{L};1)\ S_\mu(0)\}
\end{aligned}
\tag{C.12}
$$

$$
\begin{aligned}
&[F(n,1,Y,\frac{X}{L}),\alpha_\mu^+] = (\partial_Y F(n,1,Y,\frac{X}{L})\ (p_\mu + \frac{Y}{X^2}\alpha_\mu) + 2\partial_{X^2}F(n,1,Y,\frac{X}{L})\ \alpha_\mu \\
&-\frac{1}{L^2}\ \{\ \frac{YL^2}{X^2}\tilde{G}_e(n,1,Y,\frac{X}{L};1)\ \alpha_\mu - (\frac{1}{Y}\tilde{G}_e(n,0,Y,\frac{X}{L};3) \\
&+\tilde{G}_e(n,0,Y,\frac{X}{L};4))\ X^2\ p_\mu + \frac{L^2}{Y}\tilde{G}_e(n,0,Y,\frac{X}{L};2)\ S_\mu(0)\}
\end{aligned}
\tag{C.13}
$$

Finally we can easily show:

$$
[X^{2p},\alpha_\mu^+] = 2pX^{2(p-1)}\alpha_\mu. \tag{C.14}
$$

This completes all the possible commutators needed for the computations of the next Appendix.

D. Equations for I^3

In a similar manner we can work in the $I_3,\ l_{33}$ basis. We define:

$$
\begin{aligned}
a^\mu\ (D^1_{\mu\nu} + D^2_{\mu\nu})\ (I_3)^n &= \Sigma_\nu(n) \\
a^\mu\ (D^1_{\mu\nu} - D^2_{\mu\nu})\ (I_3)^n &= \Psi_\nu(n)
\end{aligned}
\tag{D.1}
$$

By induction we can show:

$$
\begin{aligned}
&[p_{1\mu} + p_{2\mu}, (I_3)^n] = -\frac{1}{L^2}\ [\frac{1}{Y} F(n-2, 0, Y, \frac{X}{L})\ (l_{31} + l_{32})\ \alpha_\mu \\
&-(F(n-2, 1, Y, \frac{X}{L}) + Y F(n-3, 1, Y, \frac{X}{L}))\ X^2\ (p_{1,\mu} + p_{2,\mu})\ ] \\
&-F(n-1, 1, Y, \frac{X}{L})\ \Psi_\nu(0)
\end{aligned}
\tag{D.2}
$$

where $Y = I_3$. We can also show that

$$
\begin{aligned}
&[p_{1\mu} - p_{2\mu}, (I_3)^n] = -\frac{1}{L^2}\ [F(n-2, 0, Y, \frac{X}{L})\ \alpha_\mu \\
&-(F(n-2, 1, Y, \frac{X}{L}) + Y F(n-3, 1, Y, \frac{X}{L}))\ X^2\ (p_{1,\mu} - p_{2,\mu})\ ] \\
&-F(n-1, 1, Y, \frac{X}{L})\ \Sigma_\nu(0)
\end{aligned}
\tag{D.3}
$$

$$
\begin{aligned}
&[(I_3)^n, \alpha^+_\mu] = n(I_3)^{n-1}(p_{1,\mu} - p_{2,\mu}) - \frac{1}{L^2}\ [\frac{Y L^2}{X^2}\ (-n\ Y^{n-1} + F(n-1, 1, Y, \frac{X}{L})\ )\ \alpha_\mu \\
&-(\frac{1}{Y}\ F(n-3, 0, Y, \frac{X}{L}) + F(n-4, 0, Y, \frac{X}{L}))\ X^2\ (p_{1,\mu} - p_{2,\mu}) \\
&+\frac{L^2}{Y}\ F(n-2, 0, Y, \frac{X}{L})\ \Sigma_\mu(0)]
\end{aligned}
$$

Actually it is fairly easy to deduce the equivalents of (C.10, C.11, C.12, C.13) for the I_3. For example the commutators of $(p_{1,\mu} - p_{2,\mu})$ with $F(n, \lambda, I_3, X)$ are deduced from (C.10, C.11) by substituting $p_{i,\mu} \to (p_{1,\mu} - p_{2,\mu})$ and $S_\mu(0) \to \Sigma_\mu(0)$. The same for the commutator of α^+_μ.

E. The vertex for an irreducible HS gauge field

In this Appendix we present the explicit computation for the BRST equations for the cubic vertex for an irreducible HS field. In order to compute (4.2) we need

$$
\begin{aligned}
&{}_{123}\langle -|\ \sum_{n+2p=s-2} (I_3)^n\ (M_{33})^p\ l_{33}\ X^2_{33;p} = \\
&{}_{123}\langle -|\ -\{(-Y^{s-2} + Y^{s-4} X^2 \begin{pmatrix} s-4 \\ s-6 \end{pmatrix})X^2_{33;0} + O(X^4)\} \times (p_{1,\mu} + p_{2,\mu})\alpha^\mu_3
\end{aligned}
\tag{E.1}
$$

$$
{}_{123}\langle -| \sum_{n+2p=s} (I_3)^n \ (M_{33})^p \ l^+_{33} X^1_p = \tag{E.2}
$$

$$
{}_{123}\langle -| - \{ \sum_{p=0}^{1} (\ (s-2p) Y^{s-2p-1}
$$

$$
+3(s-2p-2)^2 \ Y^{s-2p-3} X^2] \frac{X^{2p}}{2^p} \ X^1_p \} \times (l_0^{11} - l_0^{22})
$$

$$
+\{ \sum_{p=0}^{1} \ p \ (Y^{s-2p} - Y^{s-2p-2} X^2 \begin{pmatrix} s-2p-2 \\ s-2p-4 \end{pmatrix}) \ \frac{X^{2(p-1)}}{2^{(p-1)}} \ X^1_p
$$

$$
\sum_{p=0}^{1} (-(s-2p-1+\mathcal{D}) \begin{pmatrix} s-2p \\ s-2p-2 \end{pmatrix} - (\mathcal{D}-1) \begin{pmatrix} s-2p \\ s-2p-2 \end{pmatrix}
$$

$$
- \begin{pmatrix} s-2p \\ s-2p-3 \end{pmatrix} + 2 \begin{pmatrix} s-2p-1 \\ s-2p-2 \end{pmatrix}
$$

$$
+2 \begin{pmatrix} s-2p-2 \\ s-2p-3 \end{pmatrix}) Y^{s-2p-2} \frac{X^{2p}}{2^p} \ X^1_{s-2p,0,p}
$$

$$
+(-(s-1+\mathcal{D}) \begin{pmatrix} s \\ s-4 \end{pmatrix} - (\mathcal{D}-1) \begin{pmatrix} s \\ s-4 \end{pmatrix} - \begin{pmatrix} s \\ s-5 \end{pmatrix}
$$

$$
-(3D-7)(\begin{pmatrix} s-1 \\ s-4 \end{pmatrix} + \begin{pmatrix} s-2 \\ s-5 \end{pmatrix})
$$

$$
-4(\begin{pmatrix} s-1 \\ s-5 \end{pmatrix} + \begin{pmatrix} s-2 \\ s-6 \end{pmatrix}) Y^{s-4} X^2 \ X^1_0 \} \times (p_{1,\mu} + p_{2,\mu}) \alpha_3^\mu
$$

Finally plugging the expressions above in (4.2) and making use of (6.2) we arrive at (6.3).

References

[1] M. A. Vasiliev, Fortsch. Phys. **52**, 702 (2004) [arXiv:hep-th/0401177]. X. Bekaert, S. Cnockaert, C. Iazeolla and M. A. Vasiliev, [arXiv:hep-th/0503128]. D. Sorokin, AIP Conf. Proc. **767**, 172 (2005) [arXiv:hep-th/0405069]. N. Bouatta, G. Compere and A. Sagnotti, [arXiv:hep-th/0409068].

[2] E. S. Fradkin and M. A. Vasiliev, Nucl. Phys. B **291**, 141 (1987). E. S. Fradkin and M. A. Vasiliev, Annals Phys. **177**, 63 (1987).

[3] M. A. Vasiliev, Phys. Lett. B **243**, 378 (1990). M. A. Vasiliev, Phys. Lett. B **285**, 225 (1992). M. A. Vasiliev, Phys. Lett. B **567**, 139 (2003) [arXiv:hep-th/0304049]. M. A. Vasiliev, arXiv:0707.1085 [hep-th]. M. A. Vasiliev, JHEP **0412** (2004) 046 [arXiv:hep-th/0404124]. E. D. Skvortsov and M. A. Vasiliev, Nucl. Phys. B **756** (2006) 117 [arXiv:hep-th/0601095].

[4] E. Sezgin and P. Sundell, Nucl. Phys. B **762** (2007) 1 [arXiv:hep-th/0508158]. J. Engquist and P. Sundell, Nucl. Phys. B **752** (2006) 206 [arXiv:hep-th/0508124]. A. Sagnotti, E. Sezgin and P. Sundell, arXiv:hep-th/0501156. E. Sezgin and P. Sundell, Nucl. Phys. B **644**, 303 (2002)

[Erratum-ibid. B **660**, 403 (2003)] [arXiv:hep-th/0205131]. J. Engquist and O. Hohm, arXiv:0705.3714 [hep-th]. J. Engquist and O. Hohm, arXiv:0708.1391 [hep-th].

[5] D. Francia and A. Sagnotti, Phys. Lett. B **543**, 303 (2002) [arXiv:hep-th/0207002]. D. Francia and A. Sagnotti, Phys. Lett. B **624**, 93 (2005) [arXiv:hep-th/0507144]. D. Francia, J. Mourad and A. Sagnotti, arXiv:hep-th/0701163.

[6] I. L. Buchbinder, V. A. Krykhtin and A. Pashnev, Nucl. Phys. B **711**, 367 (2005) [arXiv:hep-th/0410215]. I. L. Buchbinder and V. A. Krykhtin, Nucl. Phys. B **727** (2005) 537 [arXiv:hep-th/0505092]. I. L. Buchbinder and V. A. Krykhtin, [arXiv:hep-th/0511276]. I. L. Buchbinder, V. A. Krykhtin, L. L. Ryskina and H. Takata, Phys. Lett. B **641** (2006) 386 [arXiv:hep-th/0603212]. I. L. Buchbinder, V. A. Krykhtin and H. Takata, arXiv:0707.2181 [hep-th]. I. L. Buchbinder, A. V. Galajinsky and V. A. Krykhtin, [arXiv:hep-th/0702161]. I. L. Buchbinder, V. A. Krykhtin and A. A. Reshetnyak, arXiv:hep-th/0703049. P. Y. Moshin and A. A. Reshetnyak, arXiv:0707.0386 [hep-th].

[7] R. R. Metsaev, Class. Quant. Grav. **11** (1994) L141. R. R. Metsaev, [arXiv:hep-th/9810231]. R. R. Metsaev, [arXiv: hep-th/0609029]. R. R. Metsaev, Phys. Lett. B **643** (2006) 205 [arXiv:hep-th/0609029].

[8] X. Bekaert, N. Boulanger and S. Cnockaert, JHEP **0601**, 052 (2006) [arXiv:hep-th/0508048]. X. Bekaert, N. Boulanger, S. Cnockaert and S. Leclercq, Fortsch. Phys. **54**, 282 (2006) [arXiv:hep-th/0602092]. N. Boulanger, S. Leclercq and S. Cnockaert, Phys. Rev. D **73** (2006) 065019 [arXiv:hep-th/0509118]. N. Boulanger and S. Leclercq, JHEP **0611** (2006) 034 [arXiv:hep-th/0609221].

[9] S. Deser and A. Waldron, Nucl. Phys. B **607** (2001) 577 [arXiv:hep-th/0103198]. S. Deser and A. Waldron, Phys. Rev. Lett. **87** (2001) 031601 [arXiv:hep-th/0102166]. S. Deser and A. Waldron, Phys. Lett. B **513** (2001) 137 [arXiv:hep-th/0105181].

[10] Yu. M. Zinoviev, Nucl. Phys. B **770** (2007) 83 [arXiv:hep-th/0609170] Yu. M. Zinoviev, [arXiv:hep-th/0108192].

[11] B. Sundborg, Nucl. Phys. Proc. Suppl. **102** (2001) 113 [arXiv:hep-th/0103247].

[12] I. R. Klebanov and A. M. Polyakov, Phys. Lett. B **550** (2002) 213 [arXiv:hep-th/0210114].

[13] A. C. Petkou, JHEP **0303** (2003) 049 [arXiv:hep-th/0302063]. R. G. Leigh and A. C. Petkou, JHEP **0306** (2003) 011 [arXiv:hep-th/0304217].

[14] K. B. Alkalaev, O. V. Shaynkman and M. A. Vasiliev, [arXiv:hep-th/0601225]. K. B. Alkalaev, O. V. Shaynkman and M. A. Vasiliev, JHEP **0508**, 069 (2005) [arXiv:hep-th/0501108].

[15] M. Bianchi, J. F. Morales and H. Samtleben, JHEP **0307**, 062 (2003) [arXiv:hep-th/0305052]. N. Beisert, M. Bianchi, J. F. Morales and H. Samtleben, JHEP **0402**, 001 (2004) [arXiv:hep-th/0310292].

[16] I. A. Bandos and J. Lukierski, Mod. Phys. Lett. A **14** (1999) 1257 [arXiv:hep-th/9811022]. I. A. Bandos, J. Lukierski and D. P. Sorokin, Phys. Rev. D **61** (2000) 045002 [arXiv:hep-th/9904109]. I. Bandos, X. Bekaert, J. A. de Azcarraga, D. Sorokin and M. Tsulaia, JHEP **0505**, 031 (2005) [arXiv:hep-th/0501113].

[17] P. West, [arXiv:hep-th/0701026].

[18] S. Fedoruk, E. Ivanov and J. Lukierski, Phys. Lett. B **641** (2006) 226 [arXiv:hep-th/0606053].

[19] A. K. H. Bengtsson, [arXiv:hep-th/0611067].

[20] P. Benincasa and F. Cachazo, arXiv:0705.4305 [hep-th].

[21] C. Fronsdal, Phys. Rev. D **18** (1978) 3624. C. Fronsdal, Phys. Rev. D **20** (1979) 848.

[22] A. K. H. Bengtsson, I. Bengtsson and L. Brink, Nucl. Phys. B **227**, 41 (1983). A. K. H. Bengtsson, Class. Quant. Grav. **5** (1988) 437.

[23] W. Siegel and B. Zwiebach, Nucl. Phys. B **263** (1986) 105.

[24] V. D. Gershun and A. I. Pashnev, Theor. Math. Phys. **73** (1987) 1227 [Teor. Mat. Fiz. **73** (1987) 294]. A. I. Pashnev, Theor. Math. Phys. **78** (1989) 272 [Teor. Mat. Fiz. **78** (1989) 384].

[25] S. Ouvry and J. Stern, Phys. Lett. B **177** (1986) 335. C. S. Aulakh, I. G. Koh and S. Ouvry, Phys. Lett. B **173** (1986) 284.

[26] X. Bekaert, I. L. Buchbinder, A. Pashnev and M. Tsulaia, Class. Quant. Grav. **21** (2004) S1457 [arXiv:hep-th/0312252].

[27] A. Neveu and P. C. West, Nucl. Phys. B **278** (1986) 601.

[28] D. J. Gross and A. Jevicki, Nucl. Phys. B **283**, 1 (1987). D. J. Gross and A. Jevicki, Nucl. Phys. B **287**, 225 (1987).

[29] A. Fotopoulos and M. Tsulaia, Phys. Rev. D **76** (2007) 025014 [arXiv:0705.2939 [hep-th]].

[30] I. L. Buchbinder, A. Fotopoulos, A. C. Petkou and M. Tsulaia, Phys. Rev. D **74** (2006) 105018 [arXiv:hep-th/0609082].

[31] D. Francia and A. Sagnotti, Class. Quant. Grav. **20** (2003) S473 [arXiv:hep-th/0212185].

[32] A. Sagnotti and M. Tsulaia, Nucl. Phys. B **682** (2004) 83 [arXiv:hep-th/0311257]. A. Fotopoulos, K. L. Panigrahi and M. Tsulaia, [arXiv:hep-th/0607248].

[33] G. Barnich, N. Bouatta and M. Grigoriev, JHEP **0510** (2005) 010 [arXiv:hep-th/0507138].

[34] X. Bekaert, arXiv:0704.0898 [hep-th].

[35] D. Anselmi, Class. Quant. Grav. **17** (2000) 1383 [arXiv:hep-th/9906167].

[36] M. A. Vasiliev, arXiv:hep-th/9910096.

[37] R. Manvelyan and W. Ruhl, Phys. Lett. B **593** (2004) 253 [arXiv:hep-th/0403241]. T. Leonhardt, R. Manvelyan and W. Ruhl, arXiv:hep-th/0401240.

[38] S. E. Konstein, M. A. Vasiliev and V. N. Zaikin, JHEP **0012**, 018 (2000) [arXiv:hep-th/0010239]. O. A. Gelfond, E. D. Skvortsov and M. A. Vasiliev, arXiv:hep-th/0601106.

[39] S. F. Prokushkin and M. A. Vasiliev, Theor. Math. Phys. **123**, 415 (2000) [Teor. Mat. Fiz. **123**, 3 (2000)] [arXiv:hep-th/9907020]. S. F. Prokushkin and M. A. Vasiliev, Phys. Lett. B **464**, 53 (1999) [arXiv:hep-th/9906149].

[40] A. Mikhailov, arXiv:hep-th/0201019.

[41] M. G. Eastwood, arXiv:hep-th/0206233.

[42] I. L. Buchbinder, V. A. Krykhtin and P. M. Lavrov, Nucl. Phys. B **762** (2007) 344 [arXiv:hep-th/0608005].

[43] I. L. Buchbinder, A. Pashnev and M. Tsulaia, Phys. Lett. B **523** (2001) 338 [arXiv:hep-th/0109067]. A. Pashnev and M. Tsulaia, Mod. Phys. Lett. A **13** (1998) 1853 [arXiv:hep-th/9803207]. I. L. Buchbinder, A. Pashnev and M. Tsulaia, [arXiv:hep-th/0206026].

[44] F. A. Berends, G. J. H. Burgers and H. van Dam, Nucl. Phys. B **271** (1986) 429. F. A. Berends, G. J. H. Burgers and H. van Dam, Nucl. Phys. B **260**, 295 (1985).

[45] H. Osborn and A. C. Petkou, Annals Phys. **231** (1994) 311 [arXiv:hep-th/9307010].